

Meson baryon components in the states of the baryon decuplet

F. Aceti,^{1,3} L. R. Dai,^{2,3} L. S. Geng,⁴ E. Oset^{1,3} and Y. Zhang²

¹*Departamento de Física Teórica and IFIC,
Centro Mixto Universidad de Valencia-CSIC,
Institutos de Investigación de Paterna,
Apartado 22085, 46071 Valencia, Spain*

²*Department of Physics,
Liaoning Normal University,
Dalian 116029, China*

³*Kavli Institute for Theoretical Physics China,
CAS, Beijing 100190, China*

⁴*Research Center for Nuclear Science and Technology
& School of Physics and Nuclear Energy Engineering,
Beihang University,
Beijing 100191, China*

(Dated: January 16, 2013)

Abstract

We apply an extension of the Weinberg compositeness condition on partial waves of $L = 1$ and resonant states to determine the amount of meson-baryon component in the $\Delta(1232)$ resonance and the other members of the $J^P = \frac{3}{2}^+$ baryon decuplet. We obtain an appreciable amount of πN in the $\Delta(1232)$ wave function, of the order of 70 %, an amount which looks more natural when one recalls that experiments on deep inelastic and Drell Yan give a fraction of πN component of 34 % for the nucleon. We also show that, as we go to higher energies in the members of the decuplet, the amount of meson-baryon component decreases and they already show a dominant part for a genuine, non meson-baryon, component in the wave function.

I. INTRODUCTION

The investigation of the structure of the different hadronic states is one of the most important topics in hadron spectroscopy. In order to describe the rich spectrum of excited hadrons quoted in the PDG [1], the traditional concept that mesons and baryons are composed, respectively, by two or three quarks, has been replaced by more complex interpretations, like the ones involving more quarks [2, 3].

A remarkable success in describing hadron structure has been obtained by chiral perturbation theory (χPT) [4, 5], an effective field theory in which the building blocks are the ground state mesons and baryons. The low energy processes are well described in this framework, but its limited energy range of convergence makes it unsuitable to deal with higher energies.

Combining unitarity constraints in coupled channels of mesons and baryons with the use of chiral Lagrangians, an extension of the theory to higher energies was made possible. The resulting theory, usually referred to as chiral unitary approach [6–18], allows to explain many mesons and baryons in terms of the meson-meson and meson-baryon interactions provided by chiral Lagrangians, interpreting them as composite states of hadrons. This kind of resonances are commonly known as “dynamically generated”.

An interesting challenge in the study of the hadron spectrum, is understanding whether a resonance can be considered as a composite state of other hadrons or else a “genuine” state. An early attempt to answer this question was made by Weinberg in a time honored work [19], in which it was determined that the deuteron was a composite state of a proton and a neutron. Other works on this issue are [20–22]. However, the analysis was made in the case of s -waves and for small binding energies. An extension to bigger binding energies, using also coupled channels and in the case of bound states, was done in [23], while in [24] also resonances are considered.

In a recent paper, the work was generalized to higher partial waves [25] and the results obtained were used to justify the commonly accepted idea that the ρ meson is not a $\pi\pi$ composite state but a genuine one. The same method was also successfully used in [26] to evaluate the amount of composite $K\pi$ state in the K^* wave function. However, no attempt was done to apply the method to baryonic resonances. We use it here to investigate the nature of the baryons of the $J^P = \frac{3}{2}^+$ decuplet.

The paper proceeds as follows. In Section II we make a brief summary of the formalism. In Section III we address the problem of πN scattering in the $\Delta(1232)$ region. In Section IV we extend the test to all the particles of the decuplet and make some conclusions in Section V.

II. OVERVIEW OF THE FORMALISM

The creation of a resonance from the interaction of many channels at a certain energy, takes place from the collision of two particles in a channel which is open.

The process is described by the set of coupled Schrödinger equations,

$$\begin{aligned} |\Psi\rangle &= |\Phi\rangle + \frac{1}{E - H_0} V |\Psi\rangle \\ &= |\Phi\rangle + \frac{1}{E - M_i - \frac{\vec{p}^2}{2\mu_i}} V |\Psi\rangle , \end{aligned} \tag{1}$$

where

$$|\Psi\rangle = \begin{Bmatrix} |\Psi_1\rangle \\ |\Psi_2\rangle \\ \vdots \\ |\Psi_N\rangle \end{Bmatrix}, \quad |\Phi\rangle = \begin{Bmatrix} |\Phi_1\rangle \\ 0 \\ \vdots \\ 0 \end{Bmatrix}, \quad (2)$$

H_0 is the free Hamiltonian and μ_i is the reduced mass of the system of total mass $M_i = m_{1i} + m_{2i}$. The state $|\Phi_1\rangle$ is the asymptotic scattering state.

Following [25], we take as the potential V

$$\langle \vec{p}|V|\vec{p}'\rangle \equiv (2l+1) v \Theta(\Lambda - p) \Theta(\Lambda - p') |\vec{p}|^l |\vec{p}'|^l P_l(\cos \theta), \quad (3)$$

where Λ is a cutoff in the momentum space and v is a $N \times N$ matrix, with N the number of channels. The form of the potential is such that the generic l -wave character of the process is contained in the two factors $|\vec{p}|^l$ and $|\vec{p}'|^l$, and in the Legendre polynomial $P_l(\cos \theta)$, so that v can be considered as a constant matrix.

The $N \times N$ scattering matrix, such that $T\Phi = V\Psi$, can be written as

$$T = (2l+1) P_l(\hat{p}, \hat{p}') \Theta(\Lambda - p) \Theta(\Lambda - p') |\vec{p}|^l |\vec{p}'|^l t, \quad (4)$$

and the Schrödinger equation leads to the Lippmann-Schwinger equation for T ($T = V + VGT$), by means of which one obtains

$$t = \frac{v}{(1 - vG)} = \frac{1}{v^{-1} - G}. \quad (5)$$

The matrix G in Eq. (5) is the loop function diagonal matrix for the two hadrons in the intermediate state (see Eq. (6)).

The derivation in [25] leads to a t matrix which does not contain the factor $|\vec{p}|^l$, since now the potential v is a constant. Differently from other approaches for p -waves, like the one of [27, 28], which factorize on shell $|\vec{p}|^l$ and associate it to the potential v , this factor is now absorbed in a new loop function

$$G_{ii} = \int_{|\vec{p}| < \Lambda} d^3p \frac{|\vec{p}|^{2l}}{E - m_{1i} - m_{2i} - \frac{\vec{p}^2}{2\mu_i}}, \quad (6)$$

which is different from the one normally used in chiral unitary approach [29].

This choice is necessary for the generalization of the sum rule for the couplings, found in [23] for the case of s -waves, to any partial wave. Indeed, as shown in [25], for a resonance or bound state dynamically generated by the interaction in coupled channels of two hadrons, the following relationship holds

$$\sum_i g_i^2 \left[\frac{dG_i}{dE} \right]_{E=E_R} = -1, \quad (7)$$

where E_R is the position of the complex pole representing the resonance and g_i is the coupling to the channel i defined as

$$g_i g_j = \lim_{E \rightarrow E_R} (E - E_R) t_{ij}. \quad (8)$$

In order to take into account the probability to have the coupling to a genuine component, Eq. (7) can be rewritten as

$$-\sum_i g_i^2 \left[\frac{dG_i}{dE} \right]_{E=E_R} = 1 - Z, \quad Z = |\langle \beta | \Psi \rangle|^2, \quad (9)$$

where $|\beta\rangle$ is the genuine component in the wave function of the state.

The left-hand side of Eq. (9) is the probability to find the hadron-hadron component, while its diversion from unity measures the probability to have something different in the wave function.

III. πN SCATTERING AND THE $\Delta(1232)$ RESONANCE

As already mentioned in the Introduction, the sum rule of Eq. (9) has been successfully applied to the ρ and K^* mesons in [25] and [26], respectively. We use it for the first time to investigate the nature of a baryonic resonance, the $\Delta(1232)$, in order to quantify the amount of πN in this state.

We first use a model based on chiral unitary theory. Then, we perform a phenomenological test which makes use only of πN scattering data and, in the last subsection, we make another estimate of the amount of genuine-state component of the Δ baryon.

A. The model dependent test

Following the approach of [25, 26] we use a potential of the type

$$v = -\alpha \left(1 + \frac{\beta}{s_R - s} \right), \quad (10)$$

where $\sqrt{s_R}$ is the bare mass of the Δ resonance and α and β are two constant factors. Note that we are putting explicitly a CDD pole in v in order to accommodate a possible genuine component of the $\Delta(1232)$ in its wave function [30]. In order to account for the p -wave character of the process, the potential v is not dependent on the momenta of the particles.

Now, we fit the πN data for the phase shifts using

$$t = \frac{1}{v^{-1} - G}. \quad (11)$$

Since the pion is relativistic in the decay of the $\Delta(1232)$, we generalize the equations as already done for the case of $\rho \rightarrow \pi\pi$ in [25]. We take only the positive energy part of the relativistic generalization of the loop function, modified to contain the $|\vec{q}|^2$ factor (see Eq. (6) and [25] for more details),

$$G(s) = \int \frac{d^3 q}{(2\pi)^3} \frac{1}{2\omega(q)} \frac{M_N}{E_N(q)} \frac{q^2}{\sqrt{s} - \omega(q) - E_N(q) + i\epsilon}, \quad (12)$$

with M_N the mass of the nucleon, m_π the mass of the pion, $E_N(q) = \sqrt{q^2 + M_N^2}$ and $\omega(q) = \sqrt{q^2 + m_\pi^2}$. The loop function in Eq. (12) is regularized by the cutoff $\theta(\Lambda - |\vec{q}|)$ of the potential (see Eq. (3)), hence Λ plays the role of q_{max} in the integral of Eq. (12).

To be more in agreement with a propagator which has a denominator linear in the energy, we slightly modify Eq. (10) as

$$v = -\frac{\alpha}{M_\Delta^4} \left(1 + \frac{\beta}{\sqrt{s_R} - \sqrt{s}} \right) , \quad (13)$$

where the factor $1/M_\Delta^4$ is introduced in order to have both parameters, α and β , in units of MeV .

The πN phase shift is given by the formula [31]

$$T = p^2 t = \frac{-4\pi\sqrt{s}}{M_N} \frac{1}{p \cot \delta(p) - ip} , \quad (14)$$

with p the momentum of the particles in the center of mass reference frame.

From the best fit to the πN data we obtain the following values of the four parameters:

$$\begin{aligned} \alpha &= 698.0 \cdot 10^3 \text{ MeV} , & \beta &= 112.5 \text{ MeV} , \\ \sqrt{s_R} &= 1313.8 \text{ MeV} , & q_{max} &= 452.6 \text{ MeV} . \end{aligned} \quad (15)$$

The results of the fit are shown in Fig. 1.

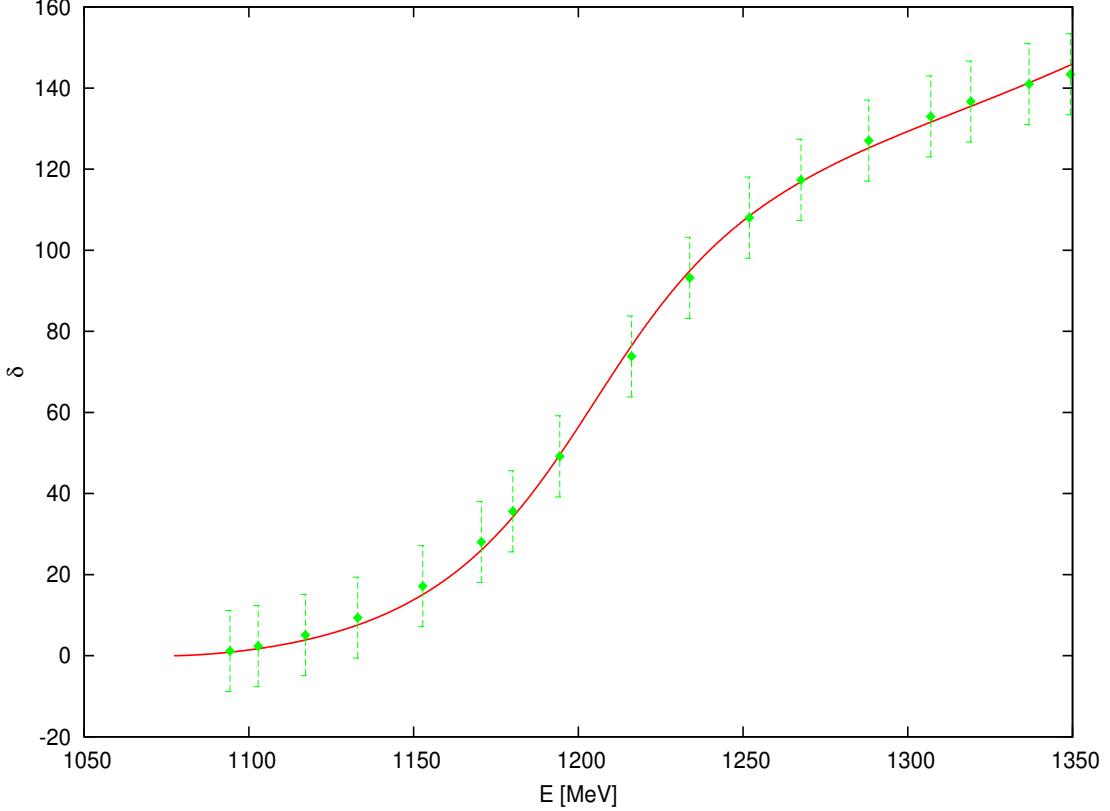


FIG. 1. The solid curve represents the πN scattering p -wave phase shifts obtained with the new approach. The data are taken from [32].

Now we want to apply the sum rule of Eq. (9) to our case. We need to extrapolate the amplitude to the complex plane and look for the complex pole $\sqrt{s_0}$ in the second Riemann

sheet. This is done by changing G to G^{II} in Eq. (11), to obtain t^{II} . The function G^{II} is the analytic continuation of the loop function in the second Riemann sheet and is defined as

$$G^{II}(s) = G^I + \frac{i}{2\pi} \frac{M_N}{\sqrt{s}} p^3, \quad Im(p) > 0, \quad (16)$$

with G^I and G^{II} the loop functions in the first and second Riemann sheet, and G^I given by Eq. (12).

We are now able to obtain the coupling \tilde{g}_Δ as the residue in the pole of the amplitude

$$\tilde{g}_\Delta^2 = \lim_{\sqrt{s} \rightarrow \sqrt{s_0}} (\sqrt{s} - \sqrt{s_0}) t^{II}, \quad (17)$$

and to apply the sum rule to evaluate the amount of the πN channel contributing to the Δ resonance

$$-\tilde{g}_\Delta^2 \left[\frac{dG^{II}(s)}{d\sqrt{s}} \right]_{\sqrt{s}=\sqrt{s_0}} = 1 - Z, \quad (18)$$

with Z the probability that the Δ is something different from a πN molecule.

The value of the pole that we get for the best fit is

$$\sqrt{s_0} = (1204.6 + i44.37) \text{ MeV}, \quad (19)$$

while for the coupling we find

$$\tilde{g}_\Delta = (8.53 + i1.85) \cdot 10^{-3} \text{ MeV}^{-1}. \quad (20)$$

From this value we finally obtain

$$1 - Z = (0.62 - i0.41), \quad (21)$$

and

$$|1 - Z| = 0.74, \quad (22)$$

which indicates a sizeable amount of πN in the resonance.

B. The phenomenological test

Now we want to evaluate the same quantity without using a model. We repeat the analysis of [25, 26] to test the sum rule by means of only the experimental data.

The Δ amplitude in a relativistic form is given by

$$t_\Delta = \frac{g_\Delta^2}{\sqrt{s} - M_\Delta + i\frac{\Gamma_{on}}{2} \left(\frac{p}{p_{on}} \right)^3}, \quad (23)$$

where

$$p = \frac{\lambda^{1/2}(s, M_N^2, m_\pi^2)}{2\sqrt{s}} \quad (24)$$

is the three-momentum of the particles in the center of mass reference frame,

$$p_{on} = p(\sqrt{s} = M_\Delta), \quad (25)$$

q_{max} [GeV]	$1 - Z$	$ 1 - Z $
0.4	$0.47 - i0.38$	0.61
0.5	$0.57 - i0.29$	0.63
0.6	$0.65 - i0.22$	0.69

TABLE I. Values of $1 - Z$ and $|1 - Z|$ for different cutoffs q_{max} .

and

$$g_{\Delta}^2 = \frac{2\pi M_{\Delta} \Gamma_{on}}{p_{on}^3 M_N} . \quad (26)$$

The values of M_{Δ} and Γ_{on} are known from the experiment.

Defining $\sqrt{s} = a + ib$ and making the substitution $p \rightarrow -p$ in the width term, we obtain the amplitude t_{Δ} in the second Riemann sheet. Then, we proceed as before to get the pole and the coupling.

The values we obtain for the pole and the coupling,

$$\begin{aligned} \sqrt{s_0} &= (1208.00 + i40.91) \text{ MeV} , \\ g_{\Delta} &= (7.78 + i1.86) \cdot 10^{-3} \text{ MeV}^{-1} , \end{aligned} \quad (27)$$

are very similar to those obtained with the model-dependent method.

Since in this case we don't know the size of the cutoff q_{max} needed to regularize the loop function, the best choice is to evaluate its derivative in Eq. (18), which in the case of p -waves is logarithmically divergent, using natural values for the cutoff to test, as done in [25, 26], the stability of the results in a certain range of q_{max} .

The values of the strength $|1 - Z|$ for three different values of q_{max} are shown in Table I. They are rather stable and consistent with the result obtained in the previous section.

C. Alternative test of the amount of genuine state.

Starting from the assumption that the $\Delta(1232)$ is a genuine state, we can make another estimate of the size of its πN component.

In this case, we take the potential to contain only the second term of Eq. (13),

$$v_{gen} = -\frac{\alpha\beta}{M_{\Delta}^4} \frac{1}{\sqrt{s_R} - \sqrt{s}} , \quad (28)$$

and the amplitude, using only this part of the potential, reads

$$t_{gen} = \frac{1}{v_{gen}^{-1} - G} = \frac{\alpha\beta}{M_{\Delta}^4} \frac{1}{\sqrt{s} - \sqrt{s_R} - G \frac{\alpha\beta}{M_{\Delta}^4}} . \quad (29)$$

Comparing Eq (29) with Eq. (23), we can write

$$g_{gen}^2 = \frac{\alpha\beta}{M_{\Delta}^4} , \quad (30)$$

which gives us the approximate value of the strength of the coupling of the resonance to the genuine component.

According to the sum rule of Eq. (18), the squared coupling of the resonance to the πN channel, that we called g_Δ , is proportional to the probability to have this component in the wave function describing the state. Analogously, g_{gen}^2 is proportional to the probability to have a genuine component. Thus, we can write

$$\frac{g_{gen}^2}{g_\Delta^2} \simeq \frac{Z}{1-Z} , \quad (31)$$

from where it follows

$$Z \simeq \frac{g_{gen}^2}{g_{gen}^2 + g_\Delta^2} . \quad (32)$$

Substituting the values of the parameters α and β obtained in Sec. III A in Eq. (30), we find that $Z \simeq 0.29$. Hence, we expect $|1 - Z|$ to be around 0.71, which is indeed the case of the values obtained in Secs. III A and III B.

Strictly speaking, $g_i G_i$ provides the Ψ_i wave function at the origin in the sharp cutoff representation of Eq. (4) in the case of s -waves (see [23, 24]), and it is proportional to the coefficient of r^l for l -waves at the origin [25]. The probabilities depend on the range of the wave functions, and hence the binding energy for each component [23]. However, for a resonance the meaning of the probabilities of the $g_i^2 \frac{\partial G_i}{\partial \sqrt{s}}$ refers to a distance in which the particles move during the finite lifetime of the resonance [33]. Thus, by construction, the range of the wave function is the same for all the components and the ratio of g_i^2 is an approximate measure of the ratio of probabilities.

IV. APPLICATION TO OTHER RESONANCES

Now we extend the study of the hadron-hadron content of resonances to the whole $J^P = \frac{3}{2}^+$ baryons decuplet.

We proceed as in the case of the $\Delta(1232)$, applying the phenomenological test of Sec. III B to the other particles of the decuplet, $\Sigma(1385)$, $\Xi(1535)$ and Ω^- .

We first investigate the $\pi\Lambda$ and $\pi\Sigma$ content of the $\Sigma(1385)$ wave function. It is known from the PDG [1] that it couples to these two channels with different branching ratios: 87% and 11.7% , respectively. In order to evaluate the couplings of the resonance to the single channel, the branching ratios must be taken into account, modifying Eq. (26) as follows:

$$g_{\Sigma^*,i}^2 = \frac{2\pi M_{\Sigma^*} \Gamma_{on}}{p_{(i)on}^3 M_i} \cdot BR^{(i)} , \quad (33)$$

where $BR^{(i)}$ is the branching ratio to the channel i , with $i = \pi\Lambda, \pi\Sigma$ and

$$p_{on}^{(i)} = p^{(i)}(\sqrt{s} = M_{\Sigma^*}) , \quad (34)$$

where

$$p^{(i)} = \frac{\lambda^{1/2}(s, M_i^2, m_\pi^2)}{2\sqrt{s}} . \quad (35)$$

On the other hand, the case of the $\Xi(1535)$ is completely analogous to the one of the $\Delta(1232)$, since, according to the PDG [1] it couples to the $\pi\Xi$ channel with a branching ratio

	channel	$\sqrt{s_0}$ [MeV]	g [MeV $^{-1}$]	$1 - Z$	$ 1 - Z $
$\Sigma(1385)$	$\pi\Lambda$	$1380.36 + i17.29$	$(5.11 + i0.60) \cdot 10^{-3}$	$(0.16 - i0.18)$	0.24
	$\pi\Sigma$	$1377.35 + i16.02$	$(3.63 + i0.81) \cdot 10^{-3}$	$(9.62 - i1.16) \cdot 10^{-2}$	0.10
$\Xi(1535)$	$\pi\Xi$	$1532.92 + i4.68$	$(4.36 + i0.23) \cdot 10^{-3}$	$0.11 - 0.09$	0.14
Ω^-	$\bar{K}\Xi$	1672.45	$(1.56 + i0.37) \cdot 10^{-2}$	0.26	0.26

TABLE II. Values of poles, couplings, $1 - Z$ and $|1 - Z|$ for the three baryons of the decuplet $J^P = \frac{3}{2}^+$, $\Sigma(1385)$, $\Xi(1535)$ and Ω^- , for a cutoff $q_{max} = 450$ MeV.

of 100%. Hence, the coupling $g_{\Xi^*,\pi\Xi}$ is simply given by Eq. (26), doing the substitutions $M_\Delta \rightarrow M_{\Xi^*}$ and $M_N \rightarrow M_\Xi$.

The case of the Ω^- is different since this resonance is stable to strong decays. This means that the on shell amplitude Γ_{on} is zero and this prevents us from evaluating the coupling of the resonance to the $\bar{K}\Xi$ channel using Eq. (26). However, from $SU(3)$ symmetry considerations we can relate the $g_{\Omega^-, \bar{K}\Xi}$ coupling to $g_{\Delta, \pi N}$, since their ratios are simply ratios of Clebsch-Gordon coefficients.

We find that

$$g_{\Omega^-, \bar{K}\Xi^0}^2 = 2g_{\Delta, \pi N}^2. \quad (36)$$

The amplitude in relativistic form is again given by Eq. (23) and, in the case of the $\Sigma(1385)$ and $\Xi(1535)$, it is extrapolated to the second Riemann sheet in order to evaluate the pole and the new couplings. Since, as already said, the Ω^- doesn't decay through strong interaction, the pole of the amplitude is found on the real axis, with no need to go to the second Riemann sheet. It is then possible to apply the sum-rule, evaluating the derivative of the G function in the position of the pole. To do it we use a cutoff of the same order of magnitude of the one found doing the best fit for the $\Delta(1232)$, $q_{max} \simeq 450$ MeV. The results obtained for the three resonances are shown in Table II.

V. DISCUSSION AND CONCLUSIONS

We have applied the generalized compositeness condition to the decuplet of the $\Delta(1232)$ to see the amount of meson-baryon cloud and genuine (presumably three quark) components. It is interesting to see that we find the pole position for the $\Delta(1232)$, Eq. (19), in very good agreement with the PDG [1] values.

As to the probability to find the πN component in the $\Delta(1232)$ wave function (with the understanding that this is in the finite extent of this component during the lifetime of the resonance), we find values which are relatively high, of the order of 70 %. This number could sound a bit large when one thinks of the $\Delta(1232)$ as just a spin flip on the quark spins of the nucleon. Yet, the result is less surprising when one recalls that from Drell Yan and deep inelastic scattering one induces a probability of about 34 % for the πN component in the nucleon [34, 35]. When one realizes this, then it also looks less surprising that, unlike the case of the ρ , where the analysis in terms of just the $\pi\pi$ component requires large counterterms beyond the lowest order contribution from the chiral Lagrangians [10, 27], in the case of the πN scattering in the $\Delta(1232)$ region a description was possible with moderate size of the counterterms [36, 37].

We extended the compositeness test to the other members of the decuplet and found a

decreasing size of the meson-baryon components when we go to the $\Sigma(1385)$ and $\Xi(1535)$, indicating that the higher energy members of the decuplet are better represented by a genuine (in principle three quark) component. For the $\Sigma(1385)$ and $\Xi(1535)$ there are also bound components of $\bar{K}N$ and $\bar{K}\Lambda$, $\bar{K}\Sigma$, respectively, which we estimate small compared to the open ones in the limited space allowed due to the decay into the open components. In the case of the Ω^- , where only the bound component $\bar{K}\Xi$ is present, we estimate the amount of the meson-baryon component to be small, of the order of 25 %.

The large pion nucleon cloud in the $\Delta(1232)$ indicates that realistic calculations of its properties should take this cloud into account. Even before the present test was done to estimate the amount of πN component in the $\Delta(1232)$ wave function, the importance of the meson cloud has been often advocated and one example of it can be seen in the early works on the cloudy bag model [38] or chiral quark model [39]. The work presented here offers a new perspective on this interesting subject and the possibility to become more quantitative than in early works.

ACKNOWLEDGMENTS

We acknowledge the hospitality of the Kavli Institute for Theoretical Physics China. This work is partly supported by the Spanish Ministerio de Economía y Competitividad and European FEDER funds under the contract number FIS2011-28853-C02-01, the Generalitat Valenciana in the program Prometeo, 2009/090, the Natural Science Foundation of China under the Grants Number 10975068, and the Project of Knowledge Innovation Program (PKIP) of the Chinese Academy of Sciences under the Grants Number KJCX2.YW.W10. F. Aceti thanks the Generalitat Valenciana for the Beca Grisolia. L.S. Geng acknowledges support from the National Natural Science Foundation of China under Grant No. 11005007. We acknowledge the support of the European Community-Research Infrastructure Integrating Activity Study of Strongly Interacting Matter (acronym HadronPhysics3, Grant Agreement n. 283286) under the Seventh Framework Programme of EU.

- [1] J. Beringer *et al.* [Particle Data Group Collaboration], Phys. Rev. D **86**, 010001 (2012).
- [2] E. Klempert and A. Zaitsev, Phys. Rept. **454**, 1 (2007) [arXiv:0708.4016 [hep-ph]].
- [3] E. Klempert and J. -M. Richard, Rev. Mod. Phys. **82**, 1095 (2010) [arXiv:0901.2055 [hep-ph]].
- [4] S. Weinberg, Physica A **96**, 327 (1979).
- [5] J. Gasser and H. Leutwyler, Annals Phys. **158**, 142 (1984).
- [6] N. Kaiser, P. B. Siegel and W. Weise, Nucl. Phys. A **594**, 325 (1995) [nucl-th/9505043].
- [7] N. Kaiser, T. Waas and W. Weise, Nucl. Phys. A **612**, 297 (1997) [hep-ph/9607459].
- [8] J. A. Oller and E. Oset, Nucl. Phys. A **620**, 438 (1997) [Erratum-ibid. A **652**, 407 (1999)] [hep-ph/9702314].
- [9] N. Kaiser, Eur. Phys. J. A **3**, 307 (1998).
- [10] J. A. Oller, E. Oset and J. R. Pelaez, Phys. Rev. D **59**, 074001 (1999) [Erratum-ibid. D **60**, 099906 (1999)] [Erratum-ibid. D **75**, 099903 (2007)] [hep-ph/9804209].
- [11] E. Oset and A. Ramos, Nucl. Phys. A **635**, 99 (1998) [nucl-th/9711022].
- [12] J. A. Oller and U. G. Meissner, Phys. Lett. B **500**, 263 (2001) [hep-ph/0011146].

- [13] D. Jido, A. Hosaka, J. C. Nacher, E. Oset and A. Ramos, Phys. Rev. C **66**, 025203 (2002) [hep-ph/0203248].
- [14] C. Garcia-Recio, J. Nieves, E. Ruiz Arriola and M. J. Vicente Vacas, Phys. Rev. D **67**, 076009 (2003) [hep-ph/0210311].
- [15] D. Jido, J. A. Oller, E. Oset, A. Ramos and U. G. Meissner, Nucl. Phys. A **725**, 181 (2003) [nucl-th/0303062].
- [16] C. Garcia-Recio, J. Nieves and L. L. Salcedo, Phys. Rev. D **74** (2006) 034025 .
- [17] T. Hyodo, S. I. Nam, D. Jido and A. Hosaka, Phys. Rev. C **68**, 018201 (2003) .
- [18] J. A. Oller, E. Oset and A. Ramos, Prog. Part. Nucl. Phys. **45**, 157 (2000) [hep-ph/0002193].
- [19] S. Weinberg, Phys. Rev. **137**, B672-B678 (1965).
- [20] C. Hanhart, Yu. S. Kalashnikova and A. V. Nefediev, Phys. Rev. D **81**, 094028 (2010) [arXiv:1002.4097 [hep-ph]].
- [21] V. Baru, J. Haidenbauer, C. Hanhart, Yu. Kalashnikova, A. E. Kudryavtsev, Phys. Lett. **B586**, 53-61 (2004).
- [22] M. Cleven, F. -K. Guo, C. Hanhart and U. -G. Meissner, Eur. Phys. J. A **47**, 120 (2011) [arXiv:1107.0254 [hep-ph]].
- [23] D. Gamermann, J. Nieves, E. Oset and E. Ruiz Arriola, Phys. Rev. D **81**, 014029 (2010) [arXiv:0911.4407 [hep-ph]].
- [24] J. Yamagata-Sekihara, J. Nieves and E. Oset, Phys. Rev. D **83**, 014003 (2011) [arXiv:1007.3923 [hep-ph]].
- [25] F. Aceti and E. Oset, Phys. Rev. D **86**, 014012 (2012) [arXiv:1202.4607 [hep-ph]].
- [26] C. W. Xiao, F. Aceti and M. Bayar, arXiv:1210.7176 [hep-ph], Eur. Phys. J. A, in print.
- [27] J. A. Oller, E. Oset and J. E. Palomar, Phys. Rev. D **63**, 114009 (2001) [hep-ph/0011096].
- [28] M. Doring and U. G. Meissner, JHEP **1201**, 009 (2012) [arXiv:1111.0616 [hep-lat]].
- [29] J. A. Oller and E. Oset, Phys. Rev. D **60**, 074023 (1999) [hep-ph/9809337].
- [30] L. Castillejo, R. H. Dalitz and F. J. Dyson, Phys. Rev. **101**, 453 (1956).
- [31] J. -J. Xie and E. Oset, Eur. Phys. J. A **48**, 146 (2012) [arXiv:1201.0149 [hep-ph]].
- [32] Center for Nuclear Studies - Data Analysis Center CNS-DAC, [Current Solution] at <http://gwdac.phys.gwu.edu/>.
- [33] T. Hyodo, D. Jido and A. Hosaka, Phys. Rev. C **85**, 015201 (2012) [arXiv:1108.5524 [nucl-th]].
- [34] W. -C. Chang and J. -C. Peng, Phys. Lett. B **704**, 197 (2011) [arXiv:1105.2381 [hep-ph]].
- [35] W. -C. Chang and J. -C. Peng, Phys. Rev. Lett. **106**, 252002 (2011) [arXiv:1102.5631 [hep-ph]].
- [36] A. Gomez Nicola, J. Nieves, J. R. Pelaez and E. Ruiz Arriola, Phys. Rev. D **69**, 076007 (2004) [nucl-th/0312034].
- [37] A. Gomez Nicola, J. Nieves, J. R. Pelaez and E. Ruiz Arriola, Phys. Lett. B **486**, 77 (2000) [hep-ph/0006043].
- [38] S. Theberge, A. W. Thomas and G. A. Miller, Phys. Rev. D **22**, 2838 (1980) [Erratum-ibid. D **23**, 2106 (1981)].
- [39] G. E. Brown, M. Rho and V. Vento, Phys. Lett. B **84**, 383 (1979).